Can hyperbolic phase of Brans-Dicke field account for Dark Matter?

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Abstract. We show that the introduction of a hyperbolic phase for Brans-Dicke (BD) field results in a flat vacuum cosmological solution of Hubble parameter H and fractional rate of change of BD scalar field, F which asymptotically approach constant values. At late stages, hyperbolic phase of BD field behaves like dark matter.

1. Introduction

It has always been one of the most challenging and interesting problems of cosmology what the composition of the universe exactly is: what was it in the primordial time and what is it in today's universe? Where did the structure of the universe originally come from? After the development of the inflationary theory [1], both observational and theoretical studies have been continuing on this subject. According to inflationary Universe models [2], inflation is capable of explaining not only the acceleration of the expansion rate but also flatness, homogeneity and isotropy of the universe. In addition, the discovery of the cosmic microwave background [3] indicates that our universe is nearly flat and expands with a slow accelerating rate [4]-[10]. This slow rate acceleration of universe results from an adequate negative pressure of dark energy and recent observations indicate that dark energy behaves like Einstein's cosmological constant [11] which arises from the vacuum energy. The remaining energy density is composed of dark matter which can not be observed directly although its gravitational effects on visible matter validate its presence. In respect of recent WMAP data [12], our universe is composed of 72 % dark energy, 23 % dark matter, and 5 % ordinary (visible) matter.

Up to now, the most popular candidate of the dark energy is the cosmological constant (vacuum energy) with the equation of state parameter $\omega = -1$. However, the observed vacuum energy density is at least 120 orders of magnitude smaller than predicted by particle physics. This is the so-called cosmological constant problem. In order to solve this problem, alternative models based on a dynamical cosmological constant Λ, with a negative equation of state have been constructed. These models include a scalar field with a slowly varying energy density. In quintessence models, the scalar field which is minimally coupled to gravity with an equation of state $\omega > -1$ acts as dark energy and a potential energy dominating over kinetic energy leads to the accelerating expansion [13]. If the scalar field has a non-canonical kinetic energy then we have k-essence models [14]. On the other hand, phantom energy models with a negative kinetic energy assert an equation of state parameter $\omega < -1$ [13]. Besides, string-theory inspired quintom models have also been analyzed. A model which includes the combination of two-scalar fields have been considered [15], one corresponding for the early time quintessence dominance, $\omega > -1$ and the other one corresponding for the late time dominance, $\omega < -1$. In addition, another string inspired quintom model where tachyon is non-minimally coupled to gravity obtained the conditions required for ω crosses over -1 [16]. Modified gravity models in the framework of scalar-tensor theories have also been analyzed to explain the acceleration of the universe [17]. A special case of these type of models is the Brans-Dicke-Jordan-Thirry [18]-[20] theory where the curvature scalar occurs only linearly in the lagrangian density. Whether the quintessence field can be identified with the Brans-Dicke-Jordan-Thirry field is an interesting question [21]-[25]. In addition to explaining dark matter, BD theory may have other advantages. In particular it has been remarked that BD theory can be

imbedded in electroweak theory [26] and it can explain the cosmic coincidence problem [30]. There exist a number of studies on accelerated models in BD theory [31]- [37]. For example, Sen *et al* [38] have found the potential relevant to power law expansion in BD cosmology. In addition, in a work of Setare [39], the lower bound of ω_Λ was found -0.9 using a holographic dark energy model in the framework of non-flat BD cosmology.

In standard cosmology the rate of expansion of the universe strongly depends on the equation of state of the matter-energy that fills it. One immediate question which arises is that whether there is any consistent modification of Einstein's equations such that the expansion of the universe is independent of its content. In the previous works of Arik, Calik and Sheftel [23]-[25], it is shown that BD scalar tensor theory of gravity with the standard mass term potential $(1/2)m^2\phi^2$ is capable of explaining the rapid primordial and slow late-time inflation and a linearized non-vacuum late time solution well accounts for the contribution of dark energy to the Friedmann Equation, however, it does not account for the contribution of dark matter. In this regard, we particularly focus on the model consists of a modified Brans-Dicke-Jordan-Thirry [18]-[20] model where both the signs of the kinetic term $(\phi\phi^* = \phi_1^2 - \phi_2^2)$ in its ϕ_2^2 part and potential term bring a minus sign. The models with this sign convention in Lagrangian have been termed as quintom models [15], [16], a word induced from quintessence and phantom. We add an imaginary part to the BD field such as $\phi = \phi_1 + i\phi_2$, and search for a contribution to dark matter in the presence of the imaginary part of ϕ field, ϕ_2.

2. Field Equations

In this work, we will show that both the dark matter contribution Ω_{DM} and dark energy contribution Ω_Λ to Friedmann Equation can be explained solely by BD theory of gravity provided that BD scalar field is modified suitably. The most straightforward modification is choosing the BD field as a complex field defined by

$$\phi = \phi_1 + i\phi_2 = \phi_{\mathrm{R}}e^{i\beta} \tag{1}$$

where ϕ_{R} is real scalar field amplitude. Such complex BD field can also be represented as in matrix form

$$\phi = \begin{pmatrix} \phi_1 & \phi_2 \\ -\phi_2 & \phi_1 \end{pmatrix} = \phi_1 + i\sigma_2\phi_2 \tag{2}$$

where σ_2 is a Pauli spin matrix.

However, we will take the phase of ϕ to be hyperbolic by replacing the term $i\beta = \Psi$ in (1) such that ϕ becomes

$$\phi = \begin{pmatrix} \phi_1 & \phi_2 \\ \phi_2 & \phi_1 \end{pmatrix} \tag{3}$$

and its conjugate matrix becomes

$$\phi^* = \begin{pmatrix} \phi_1 & -\phi_2 \\ -\phi_2 & \phi_1 \end{pmatrix} \tag{4}$$

where

$$\phi_1 = \phi_R \cosh \Psi \tag{5}$$

$$\phi_2 = \phi_R \sinh \Psi \tag{6}$$

where Ψ is real. With this modification, we note here that Ψ gains a "Quintom" character since its kinetic contribution ($\phi\phi^* = \phi_1^2 - \phi_2^2$) brings a minus sign. But nevertheless, in this paper, we will show that a cosmological vacuum solution with flat space-like section is capable of explaining how the Hubble parameter H evolves with the scale size of the universe $a(t)$ and how the solution of fractional rate of change of BD scalar field, F contributes to the evolution of H in the late era in which the universe is expanding at a slow rate. In the context of BD theory [20] with self interacting potential and matter field, the action in the canonical form for real BD scalar field in our notation is given by

$$S = \int d^4x \sqrt{g} \left[-\frac{1}{8\omega} \phi_1^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_1 \partial_\nu \phi_1 - \frac{1}{2} m^2 \phi_1^2 + L_M \right], \tag{7}$$

however, since we have modified the scalar BD field ϕ as in (4), we also modify the action above as

$$S = \frac{1}{2} \operatorname{tr} \int d^4x \sqrt{g} \left[\frac{-1}{8\omega} \phi\phi^* R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* - \frac{1}{2} m^2 \phi\phi^* + I L_M \right]. \tag{8}$$

where I is the unit matrix. In particular we may expect that ϕ is spatially uniform, but varies slowly with time. The signs of the non-minimal coupling term and the kinetic energy term are properly adopted to $(+---)$ metric signature. In units where $c = \hbar = 1$, we define Planck-length, $L_{\rm p}$, in such a way that $L_P^2 \phi_R^2 = \omega/2\pi$ where $\phi_{\rm R}$ is the present value in (5,6). Hence the dimension of the scalar field is chosen to be $L_{\rm p}^{-1}$, so that G_{eff} has a dimension L_P^2 since nonminimal coupling term $\phi_R^2 R$ where R is the Ricci scalar, replaces with the Einstein-Hilbert term $\frac{1}{G_N} R$ in such a way that $G_{eff}^{-1} = \frac{2\pi}{\omega} \phi_R^2$ where G_{eff} is the effective gravitational constant as long as the dynamical scalar field ϕ varies slowly with time. To be in accordance with the weak equivalence principle, the matter part of the Lagrangian, L_M, is decoupled from ϕ such that we have considered the energy-momentum tensor $T_\nu^\mu = diag\,(\rho, -p, -p, -p)$ just with classical perfect fluid where ρ is the energy density, p is the pressure. The gravitational field equations derived from the variation of the action (8) with respect to Robertson- Walker metric is

$$\frac{3}{4\omega} \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{2} \frac{\dot\phi\dot\phi^*}{\phi\phi^*} + \frac{3}{4\omega} \left(\frac{\dot{a}}{a} \right) \left(\frac{\dot\phi\phi^* + \phi\dot\phi^*}{\phi\phi^*} \right) - \frac{1}{2} m^2 = \frac{\rho_M}{\phi\phi^*} \tag{9}$$

$$-\frac{1}{4\omega} \left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - \left(\frac{1}{2} + \frac{1}{2\omega} \right) \frac{\dot\phi\dot\phi^*}{\phi\phi^*} - \frac{1}{2\omega} \left(\frac{\dot{a}}{a} \right) \left(\frac{\dot\phi\phi^* + \phi\dot\phi^*}{\phi\phi^*} \right) \tag{10}$$

$$-\frac{1}{4\omega} \left(\frac{\ddot\phi\phi^* + \ddot\phi^*\phi}{\phi\phi^*} \right) + \frac{1}{2} m^2 = \frac{p_M}{\phi\phi^*}$$

$$\frac{\ddot{\phi}}{\phi} + 3\left(\frac{\dot{a}}{a}\right)\frac{\dot{\phi}}{\phi} + m^2 - \frac{3}{2\omega}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = 0 \tag{11}$$

where k is the curvature parameter with $k = -1$, 0, 1 corresponding to open, flat, closed universes respectively and $a(t)$ is the scale factor of the universe (dot denotes $\frac{d}{dt}$). Since in the standard theory of gravitation, the total energy density ρ is assumed to be composed of $\rho = \rho_\Lambda + \rho_M$ where ρ_Λ is the energy density of the universe due to the cosmological constant which in modern terminology is called as *"dark energy"*, the right hand sides of (9, 11) are adopted to the matter energy density term ρ_M instead of ρ and p_M instead of p where M denotes everything except the ϕ field. The main reason behind doing such an organization is that whether if the ϕ terms on the left-hand side of (9) can accommodate a contribution to due to what is called dark matter. In addition, the right hand side of the ϕ equation (11) is set to be zero according to the assumption imposed on the matter Lagrangian L_M being independent of the scalar field ϕ. For the vacuum $(\rho = p = 0)$ and flat space like $(k = 0)$ section solutions, we start with defining the fractional rate of change of ϕ as

$$F(a) = \frac{\dot{\phi}\phi^*}{\phi\phi^*} = \begin{pmatrix} F_1 & F_2 \\ F_2 & F_1 \end{pmatrix} \tag{12}$$

where

$$F_1 = \frac{\dot{\phi}_1\phi_1 - \dot{\phi}_2\phi_2}{\phi_1^2 - \phi_2^2} = \frac{\dot{\phi}_R}{\phi_R}, \qquad F_2 = \frac{\dot{\phi}_2\phi_1 - \dot{\phi}_1\phi_2}{\phi_1^2 - \phi_2^2} = \dot{\Psi} \tag{13}$$

and the Hubble parameter as $H(a) = \dot{a}/a$, hence, we rewrite the left hand-side of the field equations (9-11) in terms of $H(a)$, $F_1(a)$, $F_2(a)$ and their derivatives with respect to the scale size of an universe a, as

$$3H^2 - 2\omega F_1^2 + 2\omega F_2^2 + 6F_1 H - 2\omega m^2 = 0 \tag{14}$$

$$3H^2 + (2\omega + 4)F_1^2 - 2\omega F_2^2 + 4F_1 H + 2aHF_1' + 2aHH' - 2\omega m^2 = 0 \tag{15}$$

$$-6H^2 + 2\omega F_1^2 + 2\omega F_2^2 + 6\omega F_1 H + 2\omega aHF_1' - 3aHH' + 2\omega m^2 = 0 \tag{16}$$

$$(4\omega F_1 + 6\omega H)F_2 + 2\omega aHF_2' = 0 \tag{17}$$

where prime denotes $\frac{d}{da}$. Since solving these coupled equations analytically is hard enough, we have put forward following perturbation solution as

$$H = H_\infty + H_1\left(\frac{a_0}{a}\right)^\alpha + H_2\left(\frac{a_0}{a}\right)^{2\alpha} \tag{18}$$

$$F_1 = F_{1\infty} + F_{11}\left(\frac{a_0}{a}\right)^\alpha + F_{12}\left(\frac{a_0}{a}\right)^{2\alpha} \tag{19}$$

$$F_2 = F_{2\infty} + F_{21}\left(\frac{a_0}{a}\right)^\alpha + F_{22}\left(\frac{a_0}{a}\right)^{2\alpha} \tag{20}$$

where H_∞, H_1, H_2, $F_{1\infty}$, F_{11}, F_{12}, $F_{2\infty}$, F_{21}, F_{22} are perturbation constants and α is an exponential factor to be determined. With the transformation

$$u = \left(\frac{a_0}{a}\right)^\alpha, \tag{21}$$

(14-17) becomes

$$3H^2 - 2\omega F_1^2 + 2\omega F_2^2 + 6HF_1 - 2\omega m^2 = 0 \tag{22}$$

$$3H^2 + (2\omega + 4)\, F_1^2 - 2\omega F_2^2 + 4HF_1 - 2\alpha u H \left(\frac{dF_1}{du}\right) - 2\alpha u H \left(\frac{dH}{du}\right) - 2\omega m^2 = 0 \tag{23}$$

$$-6H^2 + 2\omega F_1^2 + 2\omega F_2^2 + 6\omega H F_1 - 2\omega\alpha u H \left(\frac{dF_1}{du}\right) + 3\alpha u H \left(\frac{dH}{du}\right) + 2\omega m^2 = 0 \tag{24}$$

$$-2\omega\alpha u H \left(\frac{dF_2}{du}\right) + (4\omega F_1 + 6\omega H)\, F_2 = 0 \tag{25}$$

and (18-20) becomes

$$H = H_\infty + H_1 u + H_2 u^2 \tag{26}$$

$$F_1 = F_{1\infty} + F_{11} u + F_{12} u^2 \tag{27}$$

$$F_2 = F_{2\infty} + F_{21} u + F_{22} u^2. \tag{28}$$

Substituting (26-28) into (22-25) and keeping only the zeroth, first and second order terms of u and neglecting higher order terms of u, we get the following equations to be solved. In the zeroth order of u;

$$3H_\infty^2 - 2\omega F_{1\infty}^2 + 2\omega F_{2\infty}^2 + 6F_{1\infty}H_\infty - 2\omega m^2 = 0 \tag{29}$$

$$3H_\infty^2 + (2\omega + 4)\, F_{1\infty}^2 - 2\omega F_{2\infty}^2 + 4F_{1\infty}H_\infty - 2\omega m^2 = 0 \tag{30}$$

$$-6H_\infty^2 + 2\omega F_{1\infty}^2 + 2\omega F_{2\infty}^2 + 6\omega F_{1\infty}H_\infty + 2\omega m^2 = 0 \tag{31}$$

$$[4\omega F_{1\infty} + 6\omega H_\infty]F_{2\infty} = 0 \tag{32}$$

in the first order of u;

$$(6F_{1\infty} + 6H_\infty)H_1 + (6H_\infty - 4\omega F_{1\infty})F_{11} + 4\omega F_{21}F_{2\infty} = 0 \tag{33}$$

$$((6-2\alpha)H_\infty + 4F_{1\infty})H_1 + ((4-2\alpha)H_\infty + (4\omega + 8)\, F_{1\infty})F_{11} - 4\omega F_{21}F_{2\infty} = 0 \tag{34}$$

$$((3\alpha - 12)H_\infty + 6\omega F_{1\infty})H_1 + ((6\omega - 2\omega\alpha)H_\infty + 4\omega F_{1\infty})F_{11} + 4\omega F_{21}F_{2\infty} = 0 \tag{35}$$

$$[(-2\omega\alpha + 6\omega)H_\infty + 4\omega F_{1\infty}]F_{21} + 4\omega F_{11}F_{2\infty} + 6\omega H_1 F_{2\infty} = 0 \tag{36}$$

in the second order of u;

$$3H_1^2 + 6H_\infty H_2 - 2\omega F_{11}^2 - 4\omega F_{1\infty}F_{12} \tag{37}$$
$$+4\omega F_{2\infty}F_{22} + 2\omega F_{21}^2 + 6F_{1\infty}H_2 + 6F_{11}H_1 + 6F_{12}H_\infty = 0$$

$$(3 - 2\alpha)H_1^2 + (2\omega + 4)\, F_{11}^2 + (4 - 2\alpha)F_{11}H_1 + (4F_{1\infty} + 6H_\infty - 4\alpha H_\infty)H_2 \tag{38}$$
$$+[4H_\infty + (4\omega + 8)\, F_{1\infty} - 4\alpha H_\infty]F_{12} - 2\omega F_{21}^2 - 4\omega F_{2\infty}F_{22} = 0$$

$$(3\alpha - 6)H_1^2 + 2\omega F_{11}^2 + (-2\omega\alpha + 6\omega)F_{11}H_1 + (-12H_\infty + 6\alpha H_\infty + 6\omega F_{1\infty})H_2 \tag{39}$$
$$+(-4\omega\alpha H_\infty + 4\omega F_{1\infty} + 6\omega H_\infty)F_{12} + 2\omega F_{21}^2 + 4\omega F_{2\infty}F_{22} = 0$$

$$(4\omega F_{1\infty} - 4\omega\alpha H_\infty + 6\omega H_\infty)F_{22} + (-2\omega\alpha H_1 + 4\omega F_{11} + 6\omega H_1)F_{21} \tag{40}$$
$$+4\omega F_{2\infty}F_{12} + 6\omega F_{2\infty}H_2 = 0.$$

3. Solutions

Solving the equation set (29-32) and (33-36) provide respectively,

$$F_{2\infty} = 0 \qquad H_\infty = \frac{2(\omega + 1)\sqrt{\omega}m}{\sqrt{(6\omega^2 + 17\omega + 12)}} \qquad F_{1\infty} = \frac{H_\infty}{2\omega + 2} \tag{41}$$

$$\alpha = 3 + \frac{1}{\omega + 1} \qquad F_{11} = -\frac{3}{2}H_1 \qquad F_{21} = \text{free} - \text{parameter} \tag{42}$$

and afterwards, substituting (41, 42) into the equation set (37-40) yields the following equation set to be solved for H_2, F_{12}, F_{21}, F_{22} as

$$(12\omega + 18)H_\infty H_2 + (8\omega + 12)H_\infty F_{12} + 4\omega(\omega + 1)F_{21}^2 = (9\omega^2 + 21\omega + 12)H_1^2 \tag{43}$$

$$-(12\omega 16)H_\infty H_2 - (12\omega + 16)H_\infty F_{12} - 4\omega(\omega + 1)F_{21}^2 = -(9\omega^2 + 27\omega + 20)H_1^2 \tag{44}$$

$$(18\omega + 24)H_\infty H_2 - (12\omega^2 + 16\omega)H_\infty F_{12} + 4\omega(\omega + 1)F_{21}^2 = -(9\omega^2 + 21\omega + 12)H_1^2 \tag{45}$$

$$F_{22} = -\frac{H_1}{H_\infty}F_{21}. \tag{46}$$

To proceed one step further, we write the standard Friedmann equation:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_\Lambda + \Omega_M \left(\frac{a_0}{a}\right)^3 \tag{47}$$

and we fit all theory parameters to the observational density parameters;

$$\Omega_\Lambda = \frac{H_\infty^2}{H_\Sigma^2}, \tag{48}$$

$$\Omega_M = \frac{2H_\infty H_1}{H_\Sigma^2}, \tag{49}$$

where

$$H_\Sigma^2 = H_\infty^2 + 2H_\infty(H_1 + H_2) + H_1^2. \tag{50}$$

With these relations above and the constraint $\Omega_\Lambda + \Omega_M = 1$, where $\Omega_M = \Omega_{VM} + \Omega_{DM}$, we can express theoretical parameters H_1 in terms of the observational density parameters Ω_Λ, Ω_M and H_∞ as

$$H_1 = \frac{\Omega_M}{2\Omega_\Lambda}H_\infty \tag{51}$$

Using recent observational results [12] on density parameters $\Omega_{DM} \simeq 0.28$, $\Omega_\Lambda \simeq 0.72$ and $\Omega_{VM} = 0$ (since the universe we study in this theory is vacuum) together with (51) we determine;

$$H_1 = \frac{0.28}{1.44}H_\infty \simeq 0.19H_\infty. \tag{52}$$

Similarly, when we solve the equations (43-46); the solutions are;

$$H_2 H_\infty = \frac{1}{34\omega + 12\omega^2 + 24}\left(18\omega^2 H_1^2 - 8\omega^2 F_{21}^2 - 4\omega^3 F_{21}^2 - 12H_1^2 - \omega H_1^2 - 4\omega F_{21}^2 + 9\omega^3 H_1^2\right) \tag{53}$$

$$F_{12}H_\infty = \frac{1}{68\omega + 24\omega^2 + 48}\left(84H_1^2 - 4\omega^2 F_{21}^2 - 4\omega F_{21}^2 + 123\omega H_1^2 + 45\omega^2 H_1^2\right). \tag{54}$$

As $\omega \to \infty$;

$$H_2 \simeq \left(-\frac{1}{3}\omega\frac{F_{21}^2}{H_\infty} + \frac{3}{4}\omega\frac{H_1^2}{H_\infty}\right) \tag{55}$$

$$F_{12} \simeq \left(-\frac{4}{24H_\infty}F_{21}^2 + \frac{45}{24H_\infty}H_1^2\right) \tag{56}$$

$$F_{22} = -\frac{H_1}{H_\infty}F_{21} \tag{57}$$

At this point, we emphasize that H_2 must be zero in order to make sense with recent observational data on density parameters of the universe and to find the exact value for F_{21}. Therefore, we insert $H_2 = 0$ and we get

$$F_{21} \simeq 0.28H_\infty \tag{58}$$

$$F_{12} \simeq 0.05H_\infty \tag{59}$$

$$F_{22} \simeq -0.06H_\infty. \tag{60}$$

Hence, with these perturbation constants (41, 42, 52, 58-60) found from theory we can express (18-20);

$$H = H_\infty + 0.19H_\infty\left(\frac{a_0}{a}\right)^3 \tag{61}$$

$$F_1 = \frac{H_\infty}{2\omega + 2} - 0.28H_\infty\left(\frac{a_0}{a}\right)^3 + 0.05H_\infty\left(\frac{a_0}{a}\right)^6 \tag{62}$$

$$F_2 = 0.28H_\infty\left(\frac{a_0}{a}\right)^3 - 0.06H_\infty\left(\frac{a_0}{a}\right)^6 \tag{63}$$

where

$$H_\infty \simeq 0.84H_0 \tag{64}$$

if (61) is satisfied for $H = H_0$, and H_0 is the present value of the Hubble parameter [12].

4. Conclusion

In this paper, we have analyzed the dark matter Ω_{DM} and dark energy contribution Ω_Λ to Friedmann Equation solely by modified BD theory of gravitation with no other input. As far as we know, the scalar field ϕ was always examined individually, however, we brought forward a new idea such that it can have different components and each of these components can account for different energy densities.

Actually, the starting point of our motivation originates from this point in the sense that when BD theory of gravitation with solely scalar field ϕ is substituted into role as discussed in our previous work [25], we have shown that WMAP+SnIa data [12], [27]-[29]

favor this model instead of the standard Einstein cosmological model with cosmological constant (LCDM model [28]-[29]) under the condition that the new density parameter Ω_Δ induced in Friedmann equation in standard cosmology to be $\Omega_\Delta < 0$ and H_2 seen in the equation (26) to be $H_2 < 0$ instead of $H_2 = 0$. (for further information, see [25]). In other words, at this stage, we have realized that the more we force H_2 to be less than zero in the real phase of the model with individual scalar field ϕ, it fits WMAP+SnIa data much more confidently than LCDM model [28]-[29]. To do this, in the first attempt, we have used a complex scalar field $\phi = \phi_1 + i\phi_2$ so that WMAP+SnIa [12], [27]-[29] data will favor the model with modified BD field ϕ in its complex phase. Although this approach has brought brand new considerations and aspects to Friedmann Equation in the concept of dark matter and dark energy, a more suitable solution was found with the modification of scalar field by using a hyperbolic phase $i\beta = \Psi$. Having solved the field equations including the hyperbolic phase, we achieved the field equations (14-17) of modified BD scalar tensor theory namely equations of "Quintom" model.

To solve these above field equations, we put up the argument of perturbative solutions with the constant terms $H_\infty, H_1, H_2, F_{1\infty}, F_{11}, F_{12}, F_{2\infty}, F_{21}$ and F_{22}. All of these constants have made it possible to originate new predictions on dark matter and dark energy contribution of BD theory.

To begin with, the most significant evidence for the idea that this modification needs real attention is the solution of α. It can easily be seen that, when $\omega \to \infty$ (where BD approaches Einstein theory), $\alpha \to 3$, as it appears in the Friedman Equation in the form $\Omega_M \left(\frac{a_0}{a}\right)^3$. Similarly, the term H_∞ which has no scale factor term, just like the energy density term due to the cosmological constant Ω_Λ in the Friedman Equation, was found purely from theory;

$$H_\infty = \frac{2\left(\omega + 1\right)\sqrt{\omega}m}{\sqrt{\left(6\omega^2 + 17\omega + 12\right)}}. \tag{65}$$

From the equation (61) in the equation set (61-63), we see that the second term is found to be smaller than the first one and the third term is found to be smaller than the second one. Namely, the dominating term is the first one which can be interpreted as the contribution to dark energy. On the other hand, the second term can be considered as the contribution to dark matter. However, the situation is different for F_1 and F_2 as it is seen in the equations (62, 63). As it was mentioned before, in the absence of F_2 term, where $F = F_1$, the theory was able to explain the contribution to dark energy but not to dark matter. Our aim was to find a contribution to dark matter in the presence of F_2, with the component F_{21} since it is coupled with $\left(\frac{a_0}{a}\right)^3$. Namely; it is agreeable to predict that while $F_{1\infty}$ which is not coupled with a scale factor term is contributing to dark energy, F_{21} is contributing to dark matter. Hence, the introduction of a hyperbolic phase for BD field results in a flat vacuum cosmological solution of Hubble parameter H and fractional rate of change of BD scalar field, F which asymptotically approach constant values. At late stages, hyperbolic phase of BD field behaves like dark matter.

5. Acknowledgments

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